ELSEVIER Journal of Financial Markets 1 (1998) 353–383

Journal of FINANCIAL MARKETS

Aggressiveness and survival of overconfident traders

Alexandros V. Benos*

Department of Finance, HEC School of Management, F-78351 Jouy-en-Josas Cedex, France

Abstract

This paper explicitly models investor behaviour in financial markets allowing for traits linked to a notion of imperfect rationality. We study an extreme form of posterior overconfidence where some risk neutral investors overestimate the precision of their private information. They compete in market orders with another group of informed traders who have rational expectations. The participation of overconfident traders in the market leads to higher transactions volume, larger depth, more volatile and more informative prices. More importantly, such traders may make higher expected profits than rational ones and may even earn more than if they switched to rational behaviour. Their unconscious commitment to aggressive trading offers them a 'first mover advantage'. I consider an extension with risk averse market makers and find that the nature of results depends on whether exogenous noise trading exists. © 1998 Elsevier Science B.V. All rights reserved.

JEL classification: G14; D84; D82

Keywords: Overconfidence; Strategic substitutes; Precommitment

0. Introduction

Some fascinating events in financial markets, such as wide price movements with no apparent reason ('runs on the Street'), short-term profit taking and aggressive trading, have rarely been explicitly studied in theoretical finance, perhaps for the simple reason they seem dangerously close to behavioural traits of irrational traders. Some other aspects of markets which have recently been captured in the rational paradigm, (for example, trading volume and technical analysis (Blume et al., 1994; Grundy and McNichols, 1989; Brown and Jennings, 1989), leave the behavioural side out. In this paper, we try to address directly the

* E-mail: benos@hec.fr

PII S1386-4181(97)00010-4

implications on financial markets of specific human behaviour, such as overconfidence or persistent errors.

In a behavioural approach to investing, two stories are often proposed. In the first one, investors lacking important pieces of information, either natural or strategic, try to forecast them. They form probability judgments about events, about the judgments formed by other agents and so on (cf. 'beauty contest' in Keynes, 1964). The complexity of following this iteration *ad infinitum* causes the agents to stop at a certain level corresponding to their innate ability, the computational equipment at hand, the signal or action to be predicted, etc. At the higher levels of this iteration, they are, however, more likely to treat their judgments as being better than they actually are, and, in the limit, as being precisely correct. Jaffe and Winkler (1976) and Figlewski (1978) are two early examples of modeling imperfect forecasting ability in a financial economics model.

In the second story, investors receive imperfect information on asset characteristics. Some treat these signals cautiously, recognizing they contain irrelevant noise. Others think signals are perfect, thereby overestimating their precision. In the extreme case, they may consider the noisy signal received as the realization of the unknown random variable itself.

The sources of such overconfidence can be indirect, like computational constraints or frictions which, in the first story, diminish the marginal benefits of additional iterations in judgments.[1] The second story lends itself more easily to a different ad hoc cognition and decision process. For example, the trader may think he can interpret the information better than he really does (cf. 'rational beliefs' in Kurz (1994)). Stated differently, he may exhibit overconfidence in his own interpretation of information. A possible reason is a non-Bayesian non-additive probability model like the one in Gilboa and Schmeidler (1993). But, whatever the source of overconfidence, the essential point is that posterior beliefs are overoptimistic. In both stories, investors put more value on their judgments or signals than they should correctly do. Under such circumstances, we show these traders may very well survive in financial markets under some conditions.

This fallacy of excessive subjective certainty is widely documented in cognitive psychology and in the calibration literature (see Lichtenstein et al., 1982 for an excellent review). Alpert and Raiffa (1982) notice people tend to be overconfident and take on more risk than expected in many experimental contexts. Attention factors and the insensitivity of risk taking to probability estimates are also documented. Tversky and Kahneman (1974) describe anchoring behaviour in the assessment of subjective probability distributions. March and Shapira

[1] See, for example, Canning (1988) and Rubinstein (1986) on rational optimization under memory constraints.

(1987) argue that managers tend to reject estimates of risk as given to them and try to revise them. Selection and survivorship biases may also be sources of overconfidence. Successful traders usually overestimate their own contribution to their success, confusing 'brains with a bull market', against the old Wall Street adage.[2] This paper explicitly introduces such behavioural biases in the modeling of financial markets and examines their effects. The latter will, of course, depend on how the market is organized and on which participants are overconfident.

The setup draws heavily on Benos (1993). We use Kyle's (1985) one shot call auction market with many informed traders, some of which are overconfident. By construction, traders do not have common priors on asset value distribution but know about the beliefs of all their opponents. We can say they agree to disagree over priors but not over how these priors are distributed in the population. We investigate effects of overconfidence in comparison to the 'all rational traders' benchmark. Though it is hard to conceive of a market entirely comprised of overconfident traders, we can also easily accommodate an 'all overconfident traders' case for comparison purposes. Market makers compete (à la Bertrand) to service the total order flow and set a price schedule, conditional on the information they have about the order flow and conditional on their beliefs about the signals of the insiders. Exogenous liquidity traders are still necessary because, though overconfident traders behave erroneously, they still possess better information than the market maker who will not wish to deal only with traders better informed than herself.

We obtain many new and interesting results on market characteristics and traders' survival. First, under a risk neutral market maker, market depth, trading volume, price volatility and price informativeness increase when there are overconfident informed investors participating in the market. Since they put more weight on their signal than they should rationally do, they submit larger orders. This increases price volatility and informativeness since it reveals a larger part of the private signals to the public. The market maker, realizing that a part of the total order flow is due to aggressive behaviour, increases market depth.

Overconfident informed traders closely resemble Black's (1986) idea of noise trading defined as '... trading on noise as if it were information'. Indeed, we find most of his conjectures come true. Prices become more variable and markets more liquid but, contrary to Black, more efficient as well, because of risk neutrality. He notes, however, in the same paper (p. 532, footnote 7), that 'In Kyle's ... model, having more noise traders can make markets more efficient'.

---

[2] Some argue professional investors are not likely to do such suboptimal judgments, and that, in any case, compensation schemes align incentives with optimal decisions. Contrary to that, Griffin and Tversky (1992) show that experts may even be more overconfident than amateurs when the environment is hard to predict (like the financial market). Moreover, this 'compensation' argument has never been formalized and, worse, it is even some times blamed for exactly this kind of overconfidence and short-termism by portfolio managers.

More importantly, individual profits of overconfident traders may be positive and even higher than those of their rational opponents. Since rational traders fear pushing the price too much in one or the other direction, they are forced to scale down their own demands, when confronted to aggressive traders. Overconfidence becomes, therefore, an (unconscious) first-mover advantage, similar to that enjoyed by the leader in a market entry game. Aggressive trading is here analogous to overinvestment by the leader in order to reduce the follower's scale of entry (but still accommodate him). It is a barrier to mobility in the market (Dixit, 1979).

We also show that it is not in the rational traders' interest to mimic the overconfident behaviour. At an individual level, it goes against the optimality result of their Nash equilibrium strategy. But, even if we assume that everybody profiting from deviating actually mimics the overconfident aggressive trading style, such a 'group change of behaviour' is profitable only if the rational group is relatively small in size. As their number rises, the expected individual profit from deviating falls and they prefer sticking to their original Nash equilibrium strategy. This result is close to 'bandwagon behaviour' and is linked to a unimodality result on total expected profits of rational traders. We show these increase with the size of the rational group, for small values of this size, but they decrease for large group sizes. A larger number of rational traders engage in intense competition among themselves and shrink the pie which they will share. Overconfident traders as a group benefit from such competition.

Two comments are necessary at this point. First, our findings depend on the conjectures trader groups hold, not only about the distribution of uncertainty but also about each other's rationality and about the market maker's information. A Nash equilibrium can be built only if both groups have the same beliefs about the market maker's pricing schedule and about each other's strategy. Second, this paper is quite different from the DeLong et al. (1990) approach where noise traders also make positive profits. As stated in the abstract of their paper, 'the disproportionate amount of risk [noise traders] themselves create enables [them] to earn a higher expected return than rational investors'. In this paper, there are no such effects since all traders are risk neutral. Irrational investors overestimate just the precision of the signal they receive. They do not imagine asset returns following a distribution when they do not. Our result comes from a first-mover advantage, not from excessive risk taking.

We also analyze the case where the market maker is risk averse. Two equilibria are now possible depending on whether we keep exogenous uninformed liquidity traders in the model or not. If we dispense with those traders altogether, leaving the irrational, overconfident traders playing the role of 'noise traders', the latter lose money on average to the rational traders and the equilibrium changes radically. Market depth may now *fall* as the number of overconfident traders increases. On the other hand, we may keep traditional noise traders in the model. We then find all of the comparative statics results with a risk neutral market maker are still valid. To resume, risk aversion of the market maker changes results on expected profits of informed traders, depending on whether exogenous noise traders exist in the market or not. Risk aversion does not change results concerning depth, volume or price variability unless it is accompanied by a radical change of the market structure, such as eliminating uninformed traders altogether.

A few other papers deal explicitly with mistaken beliefs about risky payoffs. Odean (1996) presents three different market structures, two of which examine price-taking overconfident informed traders (like in Grossman and Stiglitz, 1980) and one which looks at a Kyle (1985) setting. His results depend a lot on the specific assumptions about risk preferences of agents and competition. In his 'Model II', which most closely resembles our market structure, results are similar with the exception of expected profits. This difference is due to the lack of competition in his model, since there is only one informed trader in the market. We can easily show the same result in the extreme case of a single overconfident insider. Caballé and Sákovics (1994) treat the question of overconfidence in a more complex setting. They assume a player's degree of overconfidence is not known to the others. So, in order to let players maintain their possibly inconsistent beliefs, they revise the solution concept of the game closer to a conjectural equilibrium. They too find higher trading volume, higher price volatility and more informative prices. Last but not least, Kyle and Wang (1997) have independently developed a similar (but more complicated) modeling of overconfidence with only two informed traders, arriving at most of our results with a risk neutral market maker. They arrive at the same result as this paper, namely that overconfident traders can make greater expected profits than rational ones, using the 'commitment to trade more aggressively' argument. Their approach, however, does not capture the competition effect between traders with different beliefs since they do not consider multiple informed traders. In this paper, we study extensively the effects of an exogenous or endogenous changes in the number of informed traders and, more specifically, we study what happens if we add more overconfident ones in the market. We find, for example, that more overconfident traders facing a market with a *given* number of rational traders increase market depth, trading volume and price variance. Moreover, we extend the model to a risk averse market maker finding that risk preferences do not change qualitatively the equilibrium.

The plan of the paper is as follows. In the following Section, we present the set-up of beliefs in detail. Then the trading game with both types of beliefs coexisting in the market is discussed and a unique Nash equilibrium with a risk neutral market maker is defined, shown to exist and characterized. In Section 3, comparisons of market depth, price volatility, trading volume and price informativeness with the two benchmark cases are presented. Section 4 discusses the survival of overconfident traders in the market. Section 5 extends the model to the case of a risk averse market maker. Section 6 concludes the paper. Proofs, when necessary, are contained in the Appendix.

1. The trading model

Consider a market for a risky asset with an ex-post liquidation value of $\tilde{F}$. There are three kinds of traders: a number of risk neutral insiders with unique access to a personalised signal $\tilde{s}_i$ about $\tilde{F}$; uninformed liquidity traders submitting a total random order $\tilde{u}$; and a competitive market maker setting a price schedule $p(\omega)$ and servicing the total order flow ω, given her beliefs about the type of insiders. The total number of informed traders is exogenous and their type decomposition (how many rational and how many overconfident types) is common knowledge.

It is also commonly known that $\tilde{s}_i = \tilde{F} + \tilde{e}_i$ where $\tilde{F}$ and $\tilde{e}_i$ are both normally distributed with means zero (for expositional reasons) and variances ϕ and ε respectively.[3] Errors $\tilde{e}_i$ are uncorrelated to all other random variables in the model. Liquidity demand has mean zero and variance of σ_u^2 and is also normally distributed.

Trading is done in two steps. First, the exogenous values of $\tilde{s}_i$ and $\tilde{u}$ are realized. Then, investors form expectations of random variables in the model, such as $\tilde{F}$; they use *belief functions* $G_i(x)$ giving, for any random variable $\tilde{x}$, the expectation of $\tilde{x}$ given i's information. To remind us of conditional expectations, we will write these belief functions as $G_i(x|s_i)$. We will call an insider *rational* if he accounts for the non-zero variance of the noise in his signal, $\tilde{e}_i$, and takes the correct mathematical conditional expectation of any random variable $\tilde{x}$. In this case, we can write

$$G_i(x|s_i) = \mathrm{E}(x|\tilde{s}_i = s_i) \tag{1}$$

We will call an investor *overconfident* if he takes the variance of $\tilde{e}_i$ as being equal to zero and uses the forecast as the actual realization of the unknown value of

[3] This 'truth plus noise' parameterization is chosen for its tractability and captures the second story described above. It is nevertheless, observationally equivalent to the first one, with the 'forecasts'. Consider the simple case with two insiders and let the underlying value $\tilde{F}$ be the sum of two uncorrelated parts $\tilde{s}_i$, each received by one insider. These signals form the strictly private information of the traders. Then, each trader forms a forecast of the signal he does not possess. Let us denote such forecast of agent i by $f_j = s_j + e_i$ $(j \neq i)$ with e_i his personalised prediction error. To summarise, each trader i observes a signal s_i and forms an estimate f_j about the signal his rival received. Trader 1's best estimate for $\tilde{F}$ is then $E_1(\tilde{F}) = s_1 + f_2 = s_1 + s_2 + e_1 = F + e_1$. Clearly, this is equal to the simpler 'truth plus noise' formulation, given the white noise garbling form used for the forecast f_j.

the stock.[4] The overconfident investor treats his forecasts as more reliable information than they really are. His belief function can be written as

$$G_i(x|s_i) = \mathrm{E}(x|\tilde{F} = s_i) \tag{2}$$

The market maker does not differ from other players in the model since she similarly forms beliefs about the distribution of her signal which is the total order flow ω. She may, for example, think (mistakenly) all traders are rational. In that case, she will be *overpricing* the order flow by extracting too much information from it. On the other hand, she may (correctly) take into account that some traders are overconfident. She will then price the order flow optimally but less than she would if she believed (or knew) all agents were rational. In what follows, we assume the market maker knows the type and number of traders and behaves optimally.

We allow both groups of rational and overconfident investors to compete against each other in quantities. This makes best responses of traders *strategic substitutes* of each other. If one trader increases the quantity of his market order, the best response of the others is, ceteris paribus, to lower their own. This plays a major role, both in the survival of overconfident traders and in market properties, as we will show in Sections 3 and 4. We can then check whether rationality proves to be a 'fitter' strategy. Although such verification has seldom been formally made (for notable exceptions, see Schaffer (1989), Blume and Easley (1992), Palomino (1996)), the 'fittest strategy' argument in support of rationality is casually used in most of finance and economics.

Both groups are aware of the presence of the other; rational traders know there are some aggressive players in the market and overconfident ones realize there are some cautious agents on the floor with them. So they differ not only in their belief about the quality of their own signal but also on their beliefs about the other group's belief. Rational traders know that $\tilde{s}_i$ is a noisy version of the payoff $\tilde{F}$ and believe that the overconfident group has received a signal of similar quality as them but treats it as the actual payoff itself. They are aware of the overconfidence of their rivals. Overconfident traders, on the other hand, believe their rivals *underestimate* the precision of their signal. They think $\tilde{s}_j$ is the realization of the payoff $\tilde{F}$, so they believe the other group treats it mistakenly as a noisy version of the payoff. In a sense, they are aware of the cautiousness of their rivals. This overconfidence is common knowledge contrary to the (first-order) beliefs about the distribution of the signal which are not. We can think of this setup not as prior overoptimistic beliefs by speculators but as evidence of

[4] This may be extreme but it is very useful in showing clearly the effects of such behaviour in financial markets. A more natural approach would be overconfident traders only underestimating slightly the error variance but this would not change the qualitative results of the paper.

posterior overconfidence. Traders think they can interpret the signal better than they really do.[5]

Since signals are not known, the game considered is a Bayesian one but without the uncertainty of types. It is common knowledge that there exists a finite number of players, M of them overconfident and N rational. Each player entertains trivial beliefs about the 'types' of other players. For example, the probability a player assigns to the event that

$$\left(\overbrace{R, \ldots, R}^{N \text{ times}}, \overbrace{O, \ldots, O}^{M-1 \text{times}}\right)$$

is the actual profile of types for the other players, is 1 if he is overconfident and 0, otherwise. This is necessary for precommitment to produce benefits for the player who has precomitted. On the contrary, such an assumption is not necessary in Schaffer (1989) since, in that model, rational agents do not modify their strategies based on the knowledge that they are playing against irrational agents. In that paper, irrational players end up obtaining higher expected utility than rational ones because they hurt rational players more than they hurt themselves (evolutionary spite). In our approach, rational agents are 'more rational' than Schaffer's (1989) profit maximisers.[6]

For each player k, the set of possible strategy profiles $X_k : s_k \mapsto \Re$, is the set of real functions from his signal to the order he submits. Purchases (sales) are positive (negative) and there are no indivisibilities. Risk neutrality and imperfect competition causes their payoff function to equal realized profits $\Pi_k(X_k, X_k) \equiv X_k(s_k)[F - p(X_k, X_{-k})]$. The (Bayesian) Nash equilibrium is defined in the usual way as the payoff maximising strategy choice.

Note that even though there may be disagreement among traders about the precision of the signal, each group forms the same first-order conjectures about the pricing schedule $p(\cdot)$ set by the market maker. Otherwise, the Nash equilibrium concept cannot be used. Caballé and Sákovics (1994) use a different concept in their approach to overconfidence. They consider a model with only overconfident traders thinking they are not overconfident even when they believe everyone else is. They consider this behaviour to be 'one of the most important characteristics' of such traders. They hence have difficulties with incorrect beliefs about strategies in equilibrium and do not use the Nash construction. In

[5] Kurz (1994) postulates economic agents do not have any structural knowledge on the underlying variables but only a large set of past data which they use to learn. Their interpretation of the same data can therefore differ vastly.

[6] This is an additional hint to the absurdity of the clairvoyance concept of Nash equilibrium. Otherwise irrational agents have consistent conjectures about the types of their rivals but an unconscious precommitment to aggressive trading; rational agents respond to such precommitment but are not selected even though they exist.

response, they develop a solution called 'Mirage' equilibrium. Players maintain possibly inconsistent beliefs but maximise against strategies which they consider to be best responses according to the beliefs they attribute to other players, and to the beliefs they think other players have about their own confidence.

A final point about common knowledge and Nash equilibrium is in order before we proceed. We need to assume that if an agent decides to change his behaviour, the other agents somehow know of the change. Such an assumption is objectionable in a static, simultaneous move game. It is, moreover, not clear that assuming consistent conjectures is a strong point when modeling irrationality. One could even interpret our result on higher overconfident expected returns as a reduction to absurdity of the Nash equilibrium concept itself. Our choice is, however, a deliberate choice of methodology. We think that, before undertaking adventurous expeditions into the lands of Nash refinements, we should be absolutely clear of the possible results of overconfidence under the most basic solution concept of Nash equilibrium.

2. Equilibrium with a risk neutral market maker

Each of the N rational insiders submits a quantity x_i and each of the M overconfident ones, submits a quantity y_j, for a total of x and y respectively. As most literature based on Kyle (1985), we characterize the unique linear Nash equilibrium. Writing $x_i = \beta s_i$ and $y_j = \delta s_j$ as the individual optimal demand functions of rational and overconfident agents respectively, each trader estimates his rivals' total demand when maximising his subjective expected profits. Rational traders use $y = \delta \sum_{j=1}^{M} s_j$ as the total demand submitted by overconfident speculators and the latter, in turn, do the corresponding estimation. Consistency is guaranteed by the assumption of common knowledge of overconfidence.

The market maker is assumed to be smart (so she prices the order flow optimally given what she knows) and risk neutral (until Section 5). She, therefore, sets prices such that she expects to earn zero profits conditional on her observing the net order flow:

$$\tilde{p}(\omega) = \mathrm{E}(F|\omega) = \lambda\tilde{\omega} \tag{3}$$

where $1/\lambda$ is the usual measure of market depth. Price linearity follows from the normality of distributions and the assumed linearity of trading strategies.

Note the market maker will choose to close the market if there are no liquidity traders. Though the overconfident traders can be thought as 'noise' traders in the sense of DeLong et al. (1991) – they 'falsely believe they have special information about the future price of the risky asset' – they still know more about the risky asset than the market maker. She will therefore lose money to both groups of insiders if there are no liquidity traders around.

Given the linear demands and the price schedule, traders solve the usual profit maximisation program. Rational trader i calculates

$$\mathrm{E}(F|s_i) = \frac{\phi}{\phi + \varepsilon} s_i , \tag{4}$$

whereas overconfident trader k lets

$$G^k(F|s_k) = s_k . \tag{5}$$

Rational traders maximize expected profits for individual demands of

$$x_i = \frac{1}{2\lambda}\frac{\phi}{\phi + \varepsilon}s_i - (N-1)\frac{\mathrm{E}(x_j|s_i)}{2} - \frac{\mathrm{E}(y|s_i)}{2}, \quad i \neq j , \tag{6}$$

while overconfident traders maximize 'perceived' profits for individual demands of

$$y_k = \frac{s_k}{2\lambda} - (M-1)\frac{G(y_l|s_k)}{2} - \frac{G(x|s_k)}{2}, \quad k \neq l . \tag{7}$$

We need to evaluate the beliefs $G(\cdot)$ of trader k for other individual overconfident orders y_l as well as his beliefs over the total demand x submitted by rational traders. In both cases, the main ingredient is k's belief about the other trader's signal, $G^k(s_l|s_k)$, which is equal to k's signal. In other words, the best estimate of a rival's signal for an overconfident trader is his own information, since he considers it to be perfect. Substituting in Eqs. (6) and (7), we get the following system:

$$\begin{aligned} &\beta + \frac{\phi M}{(N+1)\phi + 2\varepsilon}\,\delta = \frac{1}{\lambda}\frac{\phi}{(N+1)\phi + 2\varepsilon}, \\ &\frac{N}{M+1}\,\beta + \delta = \frac{1}{\lambda}\frac{1}{M+1} . \end{aligned} \tag{8}$$

Solving the above system, we have:

Lemma 1. *For a given liquidity parameter λ, the order submitted by rational trader i is βs_i and that submitted by overconfident trader k is δs_k, where*

$$\beta = \frac{1}{\lambda}\frac{\phi}{(M+N+1)\phi + 2(M+1)\varepsilon}, \tag{9}$$

$$\delta = \frac{1}{\lambda}\frac{\phi + 2\varepsilon}{(M+N+1)\phi + 2(M+1)\varepsilon} . \tag{10}$$

First, note that, for all values of market depth, overconfident speculators are using their signal more aggressively when trading. It is clear traders must have consistent conjectures (a) about the pricing parameter λ, and (b) about each other's chosen strategy. The former implies one group maximizes assuming the other is using the same conjectured pricing parameter.[7] There is certainly a large number of different conjectures about the pricing parameter traders can entertain. I study the case where both believe the market maker is pricing the order flow as coming from two distinct groups of informed traders, each with different trade intensity. The rational group thinks she is doing her job rationally (and indeed she is). The overconfident group, on the contrary, believes she prices the order flow incorrectly since she treats their information as imperfect. But they do not complain since, according to their beliefs, such behaviour is advantageous to them.

A final remark is in order here. Players maximise subjective expected profits given the equilibrium strategy of the other group *after* types have been attributed. In an extensive form game, we would have an historical chance node describing the random determination of players' types. Traders have already an identity – as rational or overconfident speculators – and then choose a profit-maximising strategy for their type among all linear strategies available.

The market maker considers all informed traders receive independently and identically distributed noisy signals of $\tilde{F}$ but a number of them treats those aggressively. She sets her market depth parameter λ equal to the regression coefficient of the payoff on the total order flow. Substituting for trade intensity coefficients, we get Lemma 2.

Lemma 2. *When the market maker is risk neutral and knows there are M overconfident and N rational informed traders in the market, a linear equilibrium exists if* $\phi^2[\phi(M+N)+\varepsilon(3M+N)]-4M\varepsilon^3 \geqslant 0$. *In this case, the liquidity parameter is given by*

$$\lambda_{rn}(M,N)=\frac{\mathrm{Cov}(F,\omega)}{\mathrm{Var}(\omega)}=\frac{\sqrt{\phi^2[\phi(M+N)+\varepsilon(3M+N)]-4M\varepsilon^3}}{\sigma_u[(M+N+1)\phi+2(M+1)\varepsilon]} \tag{11}$$

An equilibrium will not exist for large numbers of overconfident traders or for very large error variances. This confirms our intuition that overconfident traders exert a negative externality on rational ones. It is well known (Kyle

[7] We can still solve the model if the two groups do not conjecture the same lambda. For example, each group may think the market maker behaves like they do when she only behaves rationally. In their calculations, the overconfident will use a pricing schedule $p(\lambda_o)$ based on conditioning on $\tilde{s}^i=\tilde{F}$ whereas the rational will expect a pricing schedule $p(\lambda_r)$ conditioned on the (true) distribution of $s^i=\tilde{F}+e^i$. They will include the beliefs of each other in their own strategy choice and solve for both $\lambda_o=g(\lambda_o,\lambda_r)$ and $\lambda_r=h(\lambda_o,\lambda_r)$ respectively. Of course, after all, the market maker will use neither λ_o nor λ_r in her pricing schedule but a convex combination of both.

The solution concept here is definitely not a Nash equilibrium since we assume the market maker actually has the ability to form a higher level of beliefs than the traders. There is therefore no common knowledge of players' rationality.

(1985, 1989)) that (risk-neutral) insiders in an imperfectly competitive financial market face an upward-sloping supply curve and take into account possible effects of their order on the equilibrium price. They, therefore, try to reduce their impact on the price by scaling down their demands (which would have been infinite in a risk-neutral perfectly competitive setup). Unfortunately, in our case, insiders who overestimate the precision of their private information, trade heavily and risk to push the price too far up or down. Consequently, rational insiders must scale their demands down even further to compensate for the aggression of their enthusiastic colleagues. This decreases their profits since they must adapt to the strategy of the overconfident group, like Stackelberg followers. At the same time, rational traders provide a positive externality to the overconfident ones. By submitting 'cautious' demands, they accommodate their aggressive trading.

When there are too many overconfident traders in the market, rational ones cannot back down their demands any more and get out of the game. The conjectures of the overconfident investors, namely that there exist 'cautious' players submitting smaller orders than they would have done, are not satisfied any more. The market maker now faces too big an adverse selection bias (there are only aggressive informed traders in the market) and she closes down the market. In what follows, I will always assume equilibrium exists, i.e.

$$\phi^2[\phi(M+N)+\varepsilon(3M+N)]-4M\varepsilon^3>0$$

for all M, N, ϕ and ε. Note that in Eq. (11) market depth rises with σ_u and falls with ϕ. More noise trading makes the adverse selection problem less severe and market depth rises. A higher information disadvantage for the market maker (higher ϕ) decreases market depth.

3. Effects of overconfident traders

We now examine the effects of such irrational traders on market properties (such as liquidity, price variability and informativeness) and on the trade intensity of each group. We will compare equilibrium properties of a market where both types are present with those of the two extreme cases: the 'all rational case' where all informed traders are rational (this is simply a multi-agent extension of Kyle, 1985); and the 'all overconfident case' where all informed traders overestimate the precision of their information but the market maker, in fact, corrects for that in her pricing schedule. To normalize notation, equilibrium parameter x will then be denoted as a function of the number of each group, $x = x(M, N)$, with the number of overconfident traders in the market M being its first argument.

3.1. Market depth

A total order flow coming (partly) from overconfident traders is not as informative as it seems to be since overconfidence leads to more aggressive trading. Additional trading is then essentially noise and is indeed treated like that by the market maker. Facing a less severe adverse selection problem, she scales down her pricing schedule. The more overconfident insiders there are, the more she lowers her parameter, increasing market depth.

Proposition 1. *For parameter values such that the corresponding equilibria exist, overconfident traders increase market liquidity*:

$$\lambda_{rn}(2n, 0) < \lambda_{rn}(n, n) < \lambda(0, 2n), \quad n > 1. \tag{12}$$

Market depth is lowest in the 'all rational' case and highest in the 'all overconfident' one. The presence of overconfident traders increases market depth and causes rational traders to trade more. We call this effect of overconfidence on trade intensity *the market liquidity effect*. Note that an 'overconfident' market maker, that is one who believes signals received by informed participants have infinite precision, would instead reduce market depth. Hence, there is a possible link between the market maker's beliefs about the information of the traders and the depth of the market she services.

Finally, the above Proposition does not address the effect of a larger number of overconfident traders in an absolute sense since we are turning some (previously) rational informed traders into overconfident ones, keeping the total number of informed traders constant. If, instead, we increase the number of overconfident traders M keeping that of rational traders N constant (but increasing the total number of informed traders in the market), market depth obviously increases since we are just adding more informed traders in the market.

3.2. Trade intensities

We now compare trade intensities across different market environments. Recall that orders in this game are strategic substitutes, so that, when one type increases its order, the best response of the other is to decrease its own. Following the previous Section, we write $\beta(M, N)$ and $\delta(M, N)$ for the coefficients of rational and overconfident traders. Using Eqs. (9) and (10) and the pricing parameter (11), we can show the following:

Proposition 2. *For parameter values such that the corresponding equilibria exist*:

1. *If* $\phi > \varepsilon\sqrt{2}$, *then* $\delta(n, n) > \delta(2n, 0) > \beta(0, 2n) > \beta(n, n)$.
2. *If* $\phi < \varepsilon\sqrt{2}$, *then* $\delta(2n, 0) > \delta(n, n) > \beta(n, n) > \beta(0, 2n)$.

The first series of inequalities is valid when overconfidence is not extreme in the sense that the idiosyncratic error overlooked by the irrational traders is small relative to prior information ϕ. The signals received are indeed relatively precise (though do not have an infinite precision), so the degree of overconfidence is low. In this case, the strategic substitute effect dominates the market liquidity effect and the rational trader trades less aggressively than in the 'all rational' environment. His gain from a deeper market is counterbalanced by his loss from more aggressive trading. Knowing this, the overconfident insider trades more aggressively than if he is opposed to traders of his own type only. When every trader is overconfident, the positive externality of rational traders does not exist and the overconfident insider has to scale back his demand.

If, instead, the degree of overconfidence is high (case 2), the results are reversed. The aggressiveness of the overconfident trader fosters market liquidity and the rational insider trades more than if all his rivals were rational. The natural reaction of the overconfident trader is then to scale his own order down.

3.3. Trading volume

Following Admati and Pfleiderer (1988), we need to distinguish aggregate trading volume from the net total order flow, $\omega = x + y + u$ since some orders are crossed on the floor. A good measure of volume is the sum of the absolute values of each component of the order flow plus the trading done by the market maker herself.[8] So expected trading volume is

$$V(M,N) \stackrel{\text{def}}{=} V^{\mathrm{R}} + V^{\mathrm{OC}} + V^{\mathrm{L}} + V^{\mathrm{MM}}$$

or

$$V(M,N) = \mathrm{E}(|x(M,N)|) + \mathrm{E}(|y(M,N)|) + \mathrm{E}(|u|) + \mathrm{E}(|\omega(M,N)|) \tag{13}$$

identifying the contribution of each group, namely V^{R} and V^{OC} measuring the expected volume of informed traders (rational and overconfident respectively), V^{L}, that of liquidity traders and V^{MM}, that done by the market maker. Since market orders are linear in information, they have zero mean and we can measure expected trading volume by

$$V(M,N) = \mathrm{SD}[x(M,N)] + \mathrm{SD}[y(M,N)] + \mathrm{SD}[\omega(M,N)] \tag{14}$$

where $\mathrm{SD}(a)$ denotes the standard deviation of the random variable a. We drop the volume traded by noise traders since $V^{\mathrm{L}} = \sigma_u\sqrt{2/\pi}$ always. Let $V^{\mathrm{I}} = V^{\mathrm{R}} + V^{\mathrm{OC}}$ be the expected trading volume of all informed traders. It is clear

[8] This definition neglects in fact the orders crossed between traders with the same beliefs. Given the IID assumption on signals, however, such an omission is not serious.

trading volume increases with trade intensity parameters. Since overconfident insiders are using their information more aggressively than rational ones, they drive aggregate trading volume up more than if they were rational. This may partly explain anecdotal evidence of excessive volume in real markets.

Proposition 3. *For parameter values for which the corresponding equilibria exist, total trading volume is higher when overconfident traders are present in the market. More specifically,*

1. *Informed volume is higher*: $V^{\mathrm{I}}(2n, 0) > V^{\mathrm{I}}(n, n) > V^{\mathrm{I}}(0, 2n)$.
2. *The volume traded by the market maker is higher*:

$$V^{\mathrm{MM}}(2n, 0) > V^{\mathrm{MM}}(n, n) > V^{\mathrm{MM}}(0, 2n)\,.$$

The above finding shows that aggregate volume increases with the number of overconfident traders active in the market. Under such circumstances, the need of liquidity traders to produce some volume in the market is less evident.

3.4. The behaviour of prices

The market clearing price is semi-strong efficient by definition of the equilibrium and, hence, trivially unbiased. When overconfident traders are present in the market, prices are more volatile since these traders overweight their information. The market maker's correction for this aggressiveness, namely increasing depth, is not enough to compensate the excess variability in the order flow.

Higher volatility is accompanied by greater price efficiency. Through their aggressiveness, overconfident traders reveal more information to the market. In the standard Kyle (1985) approach, the informed trader tries withholding information from the market by restraining his trading activity. An overconfident trader, in contrast, offers such information to other participants by trading more intensely. He makes markets more efficient and reduces prior information asymmetry.

4. Profits and choices

In this section we examine the effects of aggressive behaviour on average profits of insiders and average losses of liquidity traders. Let $\Pi^{\mathrm{OC}}(M, N)$ ($\Pi^{\mathrm{R}}(M, N)$) be the individual realized profits of overconfident (rational) traders as a function of the size of each group. We will use the notation $G(x)$ ('perceived x') to denote expectations taken with respect to subjective probability distributions held by overconfident insiders and $\mathrm{E}(x)$ ('expected x') for the expectations taken using the true distribution of signals.

4.1. Expected vs. perceived profits

We first look at what the overconfident traders perceive as average profits. Irrational investors believe overconfidence is always optimal. This is indeed the essence of overconfidence. They believe they get higher expected returns than they would if they acted rationally. Since they also believe 'the others' underestimate the precision of their signal and they are wrong in doing so, they, being right, make more profits on average than their rivals. That is, they also believe they get higher expected returns than rational investors do. Finally, they overestimate their average profits.

Remark 1. For all admissible parameter values and for all $M, N > 1$, we find

$$G[\Pi^{\mathrm{OC}}(M,N)] > G[\Pi^{\mathrm{OC}}(M-1,N+1)]\,, \tag{15}$$

$$G[\Pi^{\mathrm{OC}}(M,N)] > G[\Pi^{\mathrm{R}}(M,N)]\,, \tag{16}$$

$$G[\Pi^{\mathrm{OC}}(M,N)] > \mathrm{E}[\Pi^{\mathrm{OC}}(M,N)]\,. \tag{17}$$

Overconfident traders believe overconfidence is always optimal. The first inequality (15) is the optimality of δ: since every type maximizes given its beliefs, its choice of strategy is optimal against the (given) strategies of the other types. The second inequality (16) shows the irrational trader thinks he will outperform his rational opponent. The final one (17) shows the irrational trader thinks he earns, on average, higher returns than he really does.

Since irrational traders maximize given their beliefs, they believe their unconditional perceived profits are positive. It is not, however, necessarily the case for their true expected profits, that is those calculated under the true distribution of signals and not the perceived one. We can show the following:

Proposition 4. *Unconditional individual expected profits are given by*

$$E[\Pi^{\mathrm{R}}(M,N)] = \frac{1}{\lambda(M,N)}\frac{\phi^2(\phi+\varepsilon)}{[(N+M+1)\phi+2(M+1)\varepsilon]^2}\,, \tag{18}$$

$$E[\Pi^{\mathrm{OC}}(M,N)] = \frac{1}{\lambda(M,N)}\frac{(\phi-\varepsilon)(\phi+2\varepsilon)^2}{[(N+M+1)\phi+2(M+1)\varepsilon]^2}\,. \tag{19}$$

Consequently, if $\phi > \varepsilon\sqrt{2}$ overconfident traders earn, on average, more profits than their rational opponents. If $\phi < \varepsilon$, they lose money on average.

We chose to write Eqs. (18) and (19) in a seemingly strange form, including an endogenous parameter like λ in the expression for expected profits, to emphasize the generality of the result which does not depend on the risk preferences of the market maker. As can be clearly seen, the relative ranking of individual expected profits among rational and irrational traders does not depend on the liquidity parameter set by the market maker, but only depends on the relative error made by irrational traders.

The above proposition shows overconfident traders may very well survive in a financial market organized as a static call auction. If the variance of their personalized error is small relative to that of the liquidation value (i.e., they do not underestimate it by a lot when they consider it equal to zero), they earn higher individual expected profits than a rational trader. They may instead lose money if the degree of overconfidence is large, that is if the error they are overlooking is significant. Note that the higher the informational disadvantage of the market maker, the lower is market depth, and, hence, the positive market liquidity effect caused by the presence of overconfident insiders becomes less important. As a result, it is more likely for irrational traders to outperform rational ones in such a case.[9]

4.2. Choosing a type

We find irrational traders are better off than rational ones unless their overconfidence is relatively strong. This result seems to contradict the optimality of the rational strategy. Since, under some circumstances, overconfident traders gain more, rational ones should have chosen to play 'overconfident' instead! This argument is nevertheless wrong because it ignores the consistency requirement of equilibrium. Rational traders have indeed the choice of playing overconfident but they consider the effects of such a choice on the distribution of participant types present in the market in equilibrium. If they choose to play aggressively, the resulting price impact of the net order flow goes up, since the number of aggressive traders in the market rises. This lowers their profits. Market orders are strategic substitutes: the derivative of one type's marginal profit with respect to its opponent's action is negative. If other traders use an aggressive strategy, my best response is to use a less aggressive strategy, and vice versa.

Their strategy is optimal given there are *already* some overconfident traders in the market. Their situation is similar to that of a follower in a Stackelberg model: once there is a leader in the market, the follower prefers being a follower than trying to lead as well.[10] If both types try to assume the role of the leader,

[9] Note the threshold level, $\varepsilon\sqrt{2}$, in Proposition 4 is the same as that in the proposition concerning trade intensities. If $\phi < \varepsilon\sqrt{2}$, the rational insider will trade more aggressively than if all insiders were rational. Such aggressiveness by the rational trader is due to the stronger market liquidity effect and causes the overconfident to scale back their demand, lowering their profits.

[10] In duopoly theory, if both firms have leader conjectures there is no equilibrium (Kreps, 1990 (Chapter 10)) due to the observability of actions. Here, actions (orders) are not directly observable by the players, so an equilibrium exists.

they will depress profits to perfect competition levels. That is not desirable for the rational traders, so they accept their role as followers.[11] They would definitely prefer being leaders themselves but that is already decided by nature in its move before the start of the game. This is summarized in the following remark; to facilitate exposition, we have input the trade coefficients chosen (β or δ) as attributes of profits:

Remark 2. If $\phi > \varepsilon\sqrt{2}$, individual expected profits satisfy the following inequalities:

$$\begin{aligned} \mathrm{E}[\Pi^{\mathrm{OC}}(M,N)] > \mathrm{E}[\Pi^{\mathrm{R}}(M,N);\beta] &\geqslant \mathrm{E}[\Pi^{\mathrm{R}}(M+1,N-1);\delta] \\ &= \mathrm{E}[\Pi^{\mathrm{OC}}(M+1,N-1)]\,. \end{aligned} \tag{20}$$

The first (strict) inequality is the result of the previous proposition. The second one is the optimality condition of β: rational traders are better off choosing β so that there are still M overconfident traders in the market, than choosing δ and mimicking the overconfident types, increasing their number from M to $M+1$. The last equality results from calculating true and not perceived profits for overconfident traders.

We can also show that there exist parameters for which the overconfident trader, not only outperforms his rational rivals (see Proposition 4) but earns higher profits than he would have had if he had traded rationally. This is a new and striking result in the literature since it means that *ex ante* it is possible for such irrational traders to gain more than rational ones.

Remark 3. For $\phi > 11\varepsilon/3$ we find $\mathrm{E}[\Pi^{\mathrm{OC}}(n,n)] > \mathrm{E}[\Pi^{\mathrm{R}}(0,2n)] > \mathrm{E}[\Pi^{\mathrm{R}}(n,n)]$

The first inequality shows that individual expected profits of an overconfident trader are higher than individual expected profits of a rational trader in the standard, 'all rationality' case. This is the crucial part of the inequality: the overconfident earns more than what he would have had if he had played the rational strategy. The second inequality simply shows these latter profits are in turn higher than the profits earned by a rational trader in a mixed environment i.e., confronted with overconfident. Note this result holds for values of ϕ, the variance of the liquidation value, which are large relatively to the signal error ε. A slightly overconfident trader (one who does not largely underestimate the

[11] In a dynamic setup, where types play the same market game repeatedly, such a strategy could probably be used as a credible threat by rationals to make speculators decrease their aggression. It would then be part of a stick-and-carrot punishment scheme.

precision of his signal when he takes it to be zero) will do better than he would if he were rational.[12]

We can interpret this advantage of overconfidence over rationality as a 'first mover's advantage'. The role of temporal asymmetry in a Spence-Dixit model of entry (Dixit (1979)) is played here by the beliefs asymmetry. Irrational traders 'burn their bridges' by playing aggressively. The paradox of commitment – common in Industrial Organization literature (see Vickers, 1985) – is applicable here. What is more interesting is the fact commitment is not a *conscious* choice. Overconfident traders do not consciously commit to being aggressive. It is only an optimal reaction given their beliefs about the precision of their private information and the type of the other participants. One might say there is no inherent difference between the two types. It is not the ad hoc truth structure (which distribution is true and which is not) which makes the difference in profits. Even if the irrational traders were indeed right (which means 'rational' traders were irrationally underestimating the precision of their signals), they would earn more than such 'overcautious' (or underconfident?) types. It is the *aggression* which buys the extra profits. Overconfidence in one's private information is just one source of aggressive behaviour.

Fishman and Hagerty (1995) show, in a completely different setting, the same commitment-to-aggression result. In their model, selling valuable information is a means for informed traders to induce other traders to trade less aggressively. In our model, it is not clear anymore that 'doing the right thing' is the optimal thing to do; overconfidence may be more profitable. Blume and Easley (1992) also show that a rational strategy might not be the fittest one in the market and, furthermore, that the fittest trading strategy might not be rational. The much cited work of DeLong et al. (1990) presents a similar result but explains the higher return earned by irrational traders to the 'bearing a disproportionate amount of risk that they themselves create…'. In this paper, overconfident insiders do not create any additional price risk. More importantly, every participant is risk neutral.[13]

The above discussion points out that, in an 'all rational' setting, whoever first becomes overconfident may outperform the other insiders who are still rational. All insiders, nevertheless, recognize this opportunity and want to be the first to exploit it. But if they all jump to the opportunity, they destroy it: $\mathrm{E}[\Pi^{\mathrm{OC}}(n,0)] < \mathrm{E}[\Pi^{\mathrm{R}}(0,n)]$.

[12] One could say that the result may change if we do consider only one overconfident trader switching to the rational strategy and not all of them as in the above remark. We can, however, show there exist parameter values for which $\mathrm{E}[\Pi^{\mathrm{OC}}(M,N)] > \mathrm{E}[\Pi^{\mathrm{OC}}(M-1,N+1)]$. Calculations are available from the author upon request.

[13] Of course, we can always interpret the exogenous 'bullishness' introduced by DeLong et al. (1990) as the consequence of the traders' overconfidence on their own model of the distribution of returns.

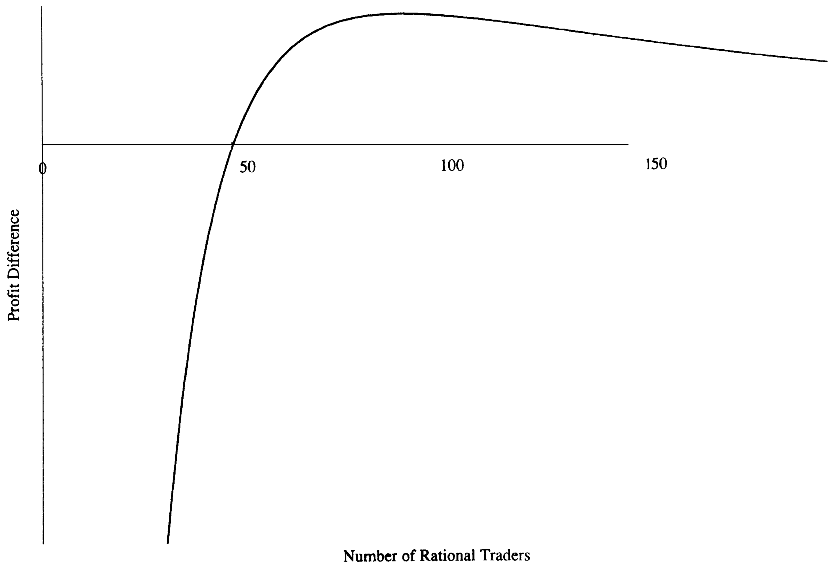

Fig. 1. Individual gains (losses) for rational traders if they all decide to deviate and follow the strategy of their overconfident rivals, as a function of N, the number of rational traders, when the market maker is risk neutral. For clarity reasons, the gain (loss) is multiplied by 1000. It is assumed that the variance of liquidity trades $\sigma_u^2 = 1$, the variance of the signal noise $\varepsilon = 2$, the variance of the underlying asset value $\phi = 5$ and the number of overconfident traders $M = 10$.

Another interesting question is whether a switch from a 'mixed' environment to the 'all irrational' case is profitable. Assume *everybody* profiting from such a deviation indeed deviates.[14] Comparing individual profits of rational traders if they all decide to deviate, $\mathrm{E}[\Pi^{\mathrm{OC}}(M+N, 0)]$, to those if they do not, $\mathrm{E}[\Pi^{\mathrm{R}}(M, N)]$, we find that such a group deviation may be profitable for some parameter values, as shown in Fig. 1. We see that if there are relatively few rational traders in the market, the most profitable strategy is to indeed deviate as a group. If their number increases, the rational strategy is preferable.

There are two probable reasons for such a behaviour. The first, most obvious one, is the size effect: the fewer they are, the more will their individual profits be by deviating. The second can be traced to the relation between group size and aggregate group profits. It is easy to show that total expected profits of overconfident traders are decreasing in the number N of rational traders while those of rational traders are unimodal in N, rising for small values

[14] This is not the same question as the optimality of β posed earlier: in calculating the best response, the trader assumes everybody else, including the other players of his own type, stick to his equilibrium prescription.

and falling for large ones. Hence, for small N, it is better for a rational trader to deviate and share the high profits of overconfident insiders. On the contrary, as N increases, total profits of overconfident traders fall rapidly while those of rational ones fall very slowly. It is better now to 'stay with the herd'. This is an example of a 'discretionary bandwagon effect'. If one is the only rational insider in the market, one prefers jumping in the bandwagon and following the overconfident herd. As group size rises, one has to share the spoils with a larger number of other deviators; it is better, therefore, not to deviate.

Consistent conjectures about opponents' moves are necessary to properly define best response reaction functions. Since an equilibrium is a pair of best response strategies, it is easy to see why traders of both types cannot deviate from their Nash prescription and increase their profits if their rivals stick to their own. It does not matter whether or not this prescription is based on erroneous first-order beliefs. We just need conjectures to be consistent so that observing each other's strategy will not make them change their own.

4.3. Liquidity traders

The market maker is assumed competitive, so she earns zero expected profits. Therefore, liquidity traders submitting random orders for consumption purposes lose exactly what informed traders gain. It is easy to see that expected losses of liquidity traders fall with market depth. So their losses fall with the presence of overconfident insiders in the market.

Note that there is a wealth transfer taking place between overconfident traders and liquidity traders. Irrational traders trade more than their rational opponents. They, hence, increase the trading volume by adding trades which are not linked to private information but are due instead to their aggressive behaviour, motivated by their erroneous beliefs. As Kyle and Wang (1997) put it, they create 'overconfident noise trading'. Such trading reduces price impact and, consequently, the expected losses of traditional noise traders. Once again, these irrational traders fall exactly into Black's (1986) definition of 'noise'.

5. A risk averse market maker

We finally consider the case of a risk averse market maker who is identical to that of the previous discussion except for the fact she possesses a negative exponential utility function with risk aversion coefficient ρ. Perfect competition with zero profits is no longer feasible since positive expected profits are needed to compensate market makers for the risks they are facing. On the other hand, if we allow risk averse market makers to compete for portions of the order flow, there are risk sharing gains to be made by splitting up the order flow in equal parts. Not to complicate the model, I follow Subrahmanyam (1991) and impose a single market maker who earns a utility of zero.[15]

The certainty equivalent obtained by the market maker when she makes the market can be written as $\mathrm{E}[\omega(p-F)|\omega]-(\rho/2)\mathrm{Var}[\omega(p-F)|\omega]$. In the unique linear equilibrium, the informed traders will follow symmetric linear strategies as the market maker uses a linear pricing rule $p=\lambda_{ra}\omega$. Substituting in the expression above and setting it equal to zero we get

$$\lambda_{ra}=\frac{E(F|\omega)}{\omega}+\frac{\rho}{2}\mathrm{Var}[F|\omega]\,. \tag{21}$$

Note that the price is no longer unbiased, so semi-strong efficiency is no longer valid. Substituting for the order flow from Section 2, we get the following proposition.

Proposition 5. (1) *When there are no exogenous liquidity traders, the equilibrium value of λ_{ra} is given by*

$$\lambda_{ra}=\frac{\rho}{2}\frac{\varepsilon\phi[N\phi^2+M(\phi+2\varepsilon)^2]}{4M\varepsilon^3-\phi^2[\phi(M+N)+\varepsilon(3M+N)]}\,. \tag{22}$$

(2) *If there exist exogenous noise traders with a demand $\tilde{u}\sim \mathrm{N}(0,\sigma_u^2)$, the equilibrium value of λ_{ra} is the unique positive real root of the cubic equation in x*:

$$2\sigma_u^2b^2x^3-\rho\phi\sigma_u^2b^2x^2+2x(a^2\phi-ab\phi+c\varepsilon)-c\rho\phi\varepsilon=0\,, \tag{23}$$

with

$$a=(N+M)\phi+2M\varepsilon\,,$$

$$b=a+\phi+2\varepsilon\,,$$

$$c=N\phi^2+M(\phi+2\varepsilon)^2\,.$$

The first part in the above proposition deals with a structure where imperfectly rational traders play the role of ‘noise traders’. We can then dispense with exogenous liquidity providers with random demands. In this case, in order for λ_{ra} to be positive, we need the denominator of the expression in (1) to be positive. This is exactly the opposite of that for the risk neutral market maker in Section 2. For the overconfident insiders to play the role of ‘noise’ traders as expected, they either need to be a very large group or they need to overestimate the precision of their signal by a lot ($\varepsilon>\phi$). In contrast, as shown in part (2), if

[15] She earns exactly what she would have earned by not making the market. This certainty equivalent is normalized to zero without loss of generality.

we choose to add an exogenous source of noise trading, like in the traditional paradigm in microstructure literature, we can guarantee equilibrium for all parameter values.

Using λ_{ra} from above, we can now easily substitute in expressions (9) and (10) to calculate the trade intensity parameters of rational and overconfident insiders, individual expected profits, group profits and the like in both cases. If there are no exogenous 'noise traders' however, overconfident informed traders lose money on average. They can not outperform their rational opponents for any parameter values. Moreover, rational insiders may increase their trade intensity as more informed traders (of any kind) enter the market. Since irrational traders play here the role of liquidity traders, we recover the standard Kyle (1985) result, namely that trading quantity increases in the uninformed order variance.

One point is nevertheless striking. With overconfident informed traders playing the role of 'noise traders', the market may lose in depth as more rational traders come in. Unlike Subrahmanyam (1991) where, with a risk averse market maker and (risk-neutral or risk averse) informed traders, the liquidity parameter λ is either unimodal or decreasing in the number of informed traders, here it may increase in N. Subrahmanyam, considers a market where all informed traders are identical, both in their beliefs and in the signal they receive. In our model, traders do not have the same beliefs about the precision of their information, nor do they receive photocopied signals. Personalized signals *add* information to the market as the number of informed traders rises. In a sense, informed traders enjoy a local monopoly but they are also in competition with each other. A risk neutral market maker focuses on the effect of competition which, like in Admati and Pfleiderer (1988), leads to a lower λ and a deeper market. On the other hand, a risk averse market maker charges a steep price schedule even if there is no informed trading because she demands compensation for bearing the risk associated with 'noise traders'. Her response function shifts up and market liquidity falls. As more information enters the market, the risks borne by the market maker increase. Therefore, she scales up her pricing schedule, reacting to an increasing number of local monopolists. For a risk averse market maker, the 'local monopoly' effect may outweigh the competition effect for a large range of parameters. Note also that, since we consider overconfident traders as 'noise traders', the increase in noise increases market depth as in the risk neutral case. Recall that, in Section 2, a higher σ_u led to a lower λ and hence, a deeper market. Fig. 2 depicts this effect.

When there exist exogenous 'liquidity traders', all of the comparative statics results stay the same as in the risk neutrality case. Market depth increases and trade intensity decrease in the number of informed traders, irrespective of their beliefs. Market depth also increases in the error variance ε and decreases in the variance of the liquidation value ϕ. Individual expected profits of both rational and overconfident trades fall in the number of traders of any group. On the contrary of the risk neutrality regime however, individual expected profits of

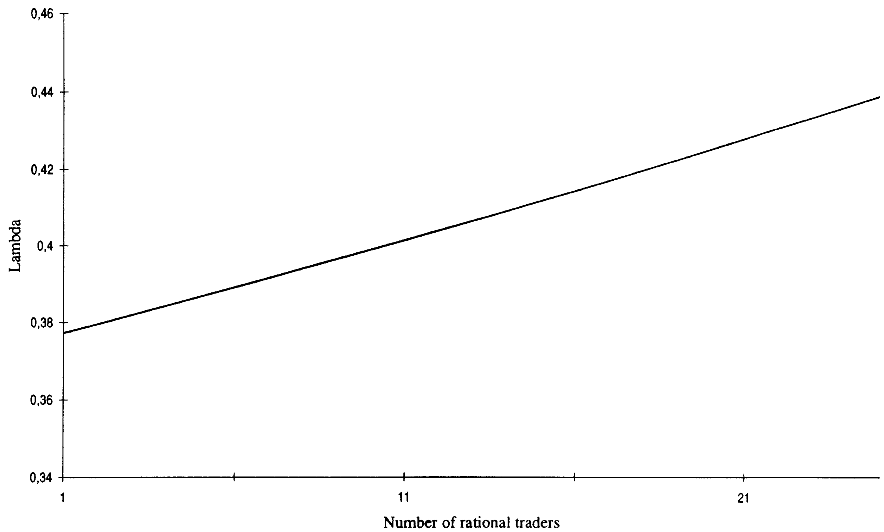

Fig. 2. Equilibrium liquidity parameter λ of a risk averse market maker as a function of the number N of rational traders. It is assumed that there are no 'pure' liquidity traders, the variance of the signal noise $\varepsilon = 1.2$, the variance of the underlying asset value $\phi = 0.5$, the risk aversion coefficient of the market maker $\rho = 1$ and the number of overconfident traders $M = 10$.

rational insiders decrease in the signal noise ε. With a risk neutral market maker this was not true: as private signals became more noisy, the degree of overconfidence increased and irrational traders started losing money which, given the zero profit condition imposed on the market maker, was transfered directly to their opponents i.e., the rational informed traders. When the market maker is risk averse, she has to be reimbursed for the risk she is facing and which rises as the private information in the market decreases in precision. Therefore, as signals become more noisy, the transfer of wealth from overconfident investors is not towards their opponents but towards the market maker. The risk premium necessary to keep her from closing the market increases.

6. Conclusion

This paper explicitly introduces behavioural traits into financial theory. It analyzes a strategic model of trading in a call auction market when some informed investors are overconfident about their estimates of unknown variables or about their ability to be 'smarter than the average'. When they compete with rational informed traders, some rather surprising effects result. First, we find such traders may make higher individual profits than their rational colleagues and survive in the market. This is due to the first-mover advantage they enjoy, precisely because they act aggressively. Commitment to an aggressive strategy in a game where actions are strategic substitutes earns higher profits and questions the ubiquitous – but implicit – assumption of the payoff superiority of a rational action. Second, they increase market depth, price variability, informativeness and volume when the market maker is risk neutral. Overconfident informed traders generate additional trading which is close to Black's (1986) discussion on noise. Besides verifying most of Black's conjectures on the effects of noise on market properties, their presence reduces expected losses of liquidity traders, precisely those who have traditionally been used to provide a cover for informed insiders. In contrast, when the market maker is risk averse, we cannot dispense with exogenous liquidity traders and market depth is not monotone any more.

In that incomplete information game, the precise conjectures that trader types hold about each other's rationality and about the market maker's beliefs are important. Common knowledge of each other's behaviour is necessary for the equilibrium to be well defined. We assume all market participants know about the overconfidence of some traders and react accordingly. Similarly, these overconfident traders know about the cautiousness of a group of their opponents and adapt their trading strategies in turn.

Introducing explicit patterns in traders' behaviour sheds some new light into market phenomena like 'irrational' traders' persistence in the market and high price volatility. It also allows to compare directly the relative 'fitness' of strategies with, sometimes, surprising results. Overconfidence is, admittedly, only one of many deviations away from fully rational behaviour. It seems, however, that such specific deviations can give interesting interpretations to events encountered in actual markets and offer an alternative approach to microstructure modeling.

This paper is only a first approach to possible modeling of behavioural traits in finance. Open questions, such as persistence of overconfidence through time, and learning, still remain. For example, it would be an interesting extension for the misperception of insiders to vary inversely with the quality of their signal: when their signal is precise enough, they treat it as a perfect one; when its quality is mediocre they only overestimate it. It may also be fruitful to examine issues such as information markets, postearnings announcement price drifts and market manipulation under these different behaviour patterns. Explicit modeling of non-Bayesian probability updating has also been noted as a possible source of chaotic behaviour in security markets but no concrete results have been produced as far as we know.[16] Bounded rationality, alias behavioural research,

[16] For a good start on such exotic paths, see the Proceedings volume of the interdisciplinary conference at the Santa Fe Institute on Complex Systems and especially the article by Brock (1988).

in finance and economics can be promising if it can provide an exposition of the exact way in which economic agents err and if it accounts in detail for possible implications of these deviations from full rationality.

Acknowledgements

Financial support from the HEC School of Management is greatefully acknowledged. I thank Anat Admati, Pierre Collin-Dufresne, Darrell Duffe, Thierry Foucault, Philippe Henrotte, Jószef Sákovics, Avanidhar Subrahmanyam (the editor), and Andrei Shleifer for their helpful comments. Frédéric Pannecière provided excellent research assistance. I also benefited from comments by an anonymous referee and seminar participants at the 7th World Congress and the 6th European Meeting of the Econometric Society. All remaining errors are mine.

Appendix A.

A.1. Proofs

Proof of Proposition 1. Since Lemma 2 gives the value of $\lambda_{rn}(M, N)$ as a function of the size of the two groups, we can compare market depth in our case with the two benchmark cases of 'all rational' and 'all overconfident' traders. By direct substitution, we can write

$$\lambda_{rn}(2n, 0) = \frac{\sqrt{2n}\sqrt{\phi - \varepsilon}}{\sigma_u(2n+1)} \tag{A.1}$$

$$\lambda_{rn}(n, n) = \frac{\sqrt{2n}\sqrt{\phi^2(\phi + 2\varepsilon) - 2\varepsilon^3}}{\sigma_u[(2n+1)\phi + 2\varepsilon(n+1)]} \tag{A.2}$$

$$\lambda_{rn}(0, 2n) = \frac{\phi\sqrt{2n}\sqrt{\phi + \varepsilon}}{\sigma_u[(2n+1)\phi + 2\varepsilon]} \tag{A.3}$$

We first need to impose a condition on ϕ and ε to ensure that all liquidity coefficients are well defined. For $\lambda_{rn}(n, n)$, ϕ has to be larger than the unique real root of $\phi^2(\phi + 2\varepsilon) - 2\varepsilon^3 = 0$. For $\lambda_{rn}(2n, 0)$, we need $\phi > \varepsilon$. The first condition is, therefore, more stringent.

Overconfident traders will increase liquidity if Eq. (A.2) is smaller than Eq. (A.3) and larger than Eq. (A.1). By squaring and cross multiplying, we find

$$\lambda_{rn}(n, n) < \lambda_{rn}(0, 2n) \quad \forall n\,, \tag{A.4}$$

$$\lambda_{rn}(2n, 0) < \lambda_{rn}(n, n) \quad \text{if } \phi > \varepsilon\,. \tag{A.5}$$

Both inequalities are always true since for equilibrium to exist in the 'all overconfident' case, we need $\phi > \varepsilon$. □

Proof of Proposition 2. In Eqs. (9) and (10) defining demand coefficients, we can substitute for the liquidity parameter λ_{rn} from above (Eqs. (A.1), (A.2) and (A.3)) to get the following:

$$\delta(2n, 0) = \frac{\sigma_u}{\sqrt{2n}\sqrt{\phi - \varepsilon}}, \tag{A.6}$$

$$\delta(n, n) = \frac{\sigma_u(\phi + 2\varepsilon)}{\sqrt{2n}\sqrt{\phi^2(\phi + 2\varepsilon) - 2\varepsilon^3}}, \tag{A.7}$$

$$\beta(n, n) = \frac{\sigma_u \phi}{\sqrt{2n}\sqrt{\phi^2(\phi + 2\varepsilon) - 2\varepsilon^3}}, \tag{A.8}$$

$$\beta(0, 2n) = \frac{\sigma_u}{\sqrt{2n}\sqrt{\phi + \varepsilon}}. \tag{A.9}$$

It is obvious $\beta(0, 2n) = \delta(0, 2n)$ and $\beta(2n, 0) = \delta(2n, 0)$ in the two benchmark cases. We also know that $\delta(n, n) > \beta(n, n)$ for all parameter values where equilibrium exists. We can then easily show:

1. $\delta(2n, 0) > \beta(0, 2n)$.
2. $\beta(0, 2n) > \beta(n, n)$, for $\phi > \varepsilon\sqrt{2}$ and $\beta(0, 2n) < \beta(n, n)$, otherwise.
3. $\delta(2n, 0) < \delta(n, n)$, for $\phi > \varepsilon\sqrt{2}$ and $\delta(2n, 0) > \delta(n, n)$, otherwise.

We combine items 1, 2 and 3 to get the result of the proposition. □

Proof of Proposition 3. From the definition of informed volume, we can write

$$V^{\mathrm{I}}(2n, 0) = \delta(2n, 0)\sqrt{2n(2n\phi + \varepsilon)} = \sigma_u \sqrt{\frac{2n\phi + \varepsilon}{\phi - \varepsilon}},$$

$$V^{\mathrm{I}}(n, n) = [\beta(n, n) + \delta(n, n)]\sqrt{n(n\phi + \varepsilon)} = \sigma_u(\phi + \varepsilon)\, sqrt \frac{2(n\phi + \varepsilon)}{\phi^2(\phi + 2\varepsilon) - 2\varepsilon^3},$$

$$V^{\mathrm{I}}(0, 2n) = \beta(0, 2n)\sqrt{2n(2n\phi + \varepsilon)} = \sigma_u \sqrt{\frac{2n\phi + \varepsilon}{\phi + \varepsilon}}.$$

It is clear that $V^{\mathrm{I}}(2n,0) > V^{\mathrm{I}}(n,n) > V^{\mathrm{I}}(0,2n)$. For the market maker volume, we use the definition of the total order flow to calculate the following:

$$V^{\mathrm{MM}}(M,N) = \sqrt{\operatorname{Var}\omega} = \sqrt{(N\beta + M\delta)^2\phi + (N\beta^2 + M\delta^2)\varepsilon + \sigma_u^2}$$

$$= \sigma_u \sqrt{\frac{\phi[\phi(N+M) + 2M\varepsilon]^2 + \varepsilon[N\phi^2 + M(\phi + 2\varepsilon)^2]}{\phi^2[\phi(N+M) + (3M+N)\varepsilon] - 4M\varepsilon^3} + 1}\,.$$

Since we restrain parameters so that equilibrium exists in all cases (i.e., $\phi^2(\phi + 2\varepsilon) - 2\varepsilon^3 > 0$), it is then easy to show $V^{\mathrm{MM}}(2n,0) > V^{\mathrm{MM}}(n,n) > V^{\mathrm{MM}}(0,2n)$. $\square$

A.2. Profits and losses

We start by calculating individual realized profits, $\Pi_k(M,N)$ of agent k

$$\Pi_i^R(M,N) = \beta(M,N)s_i H\,, \tag{A.10}$$

$$\Pi_i^{\mathrm{OC}}(M,N) = \delta(M,N)s_i H\,, \tag{A.11}$$

where

$$H = [1 - \lambda_{rn}(M,N)][\beta(M,N) + \delta(M,N)]\tilde{F} - \lambda_{rn}(M,N)$$
$$\left[\beta(M,N)\sum_i^N \tilde{e}_i + \delta(M,N)\sum_j^M \tilde{e}_j\right] - \lambda_{rn}(M,N)\tilde{u}\,. \tag{A.12}$$

In order to take conditional expectations of H with respect to beliefs held by overconfident traders, we use the fact that $G(\tilde{e}_k|s_k) = 0$, for all k and that $G(\tilde{F}|s_k) = s_k$. After some algebraic manipulations, we find:

$$G[\Pi^{\mathrm{OC}}(M,N)] = \delta(M,N)\frac{\phi(\phi + 2\varepsilon)}{(N+M+1)\phi + 2(M+1)\varepsilon}, \tag{A.13}$$

$$G[\Pi^{\mathrm{R}}(M,N)] = \beta(M,N)\frac{\phi(\phi + 2\varepsilon)}{(N+M+1)\phi + 2(M+1)\varepsilon}, \tag{A.14}$$

Since $\delta(M,N) > \beta(M,N)$, we get the second inequality in Remark 1. The third inequality needs computing expected profits of an overconfident insider which equal

$$\mathrm{E}[\Pi^{\mathrm{OC}}(M,N)] = \delta(M,N)\frac{(\phi - \varepsilon)(\phi + 2\varepsilon)}{(N+M+1)\phi + 2(M+1)\varepsilon}$$
$$< G[\Pi^{\mathrm{OC}}(M,N)]\,, \tag{A.15}$$

which proves the second equality in Proposition 4. Calculating the expected profits of a rational trader is straightforward and yields

$$\mathrm{E}[\Pi^{\mathrm{R}}(M,N)] = \beta(M,N)\frac{\phi(\phi+\varepsilon)}{(N+M+1)\phi+2(M+1)\varepsilon}\,. \tag{A.16}$$

The rest of Proposition 4 follows immediately. □

A.3. Risk averse market maker

If there are no exogenous noise traders, we have

$$\mathrm{Cov}(F,\omega) = AB\phi\lambda_{\mathrm{ra}}\,, \tag{A.17}$$

$$\mathrm{Var}(\omega) = A^2\phi + C\varepsilon\,, \tag{A.18}$$

$$\mathrm{Var}(F|\omega) = \frac{C\varepsilon\phi}{A^2\phi + C\varepsilon}\,, \tag{A.19}$$

where constants are $A=(N+M)\phi+2M\varepsilon$, $B=(N+M+1)\phi+2(M+1)\varepsilon$ and $C=N\phi^2+M(\phi+2\varepsilon)^2$. The expression for λ_{ra} then follows by substitution.

When exogenous noise traders are present, variances change:

$$\mathrm{Var}(\omega) = A^2\phi + C\varepsilon + B^2\sigma_u^2\lambda_{\mathrm{ra}}^2\,, \tag{43}$$

$$\mathrm{Var}(F|\omega) = \phi\,\frac{C\varepsilon + B^2\sigma_u^2\lambda_{\mathrm{ra}}^2}{A^2\phi + C\varepsilon + B^2\sigma_u^2\lambda_{\mathrm{ra}}^2}\,. \tag{44}$$

Using the definition in Eq. (21), we get a third-order equation in λ_{ra}:

$$f(\lambda_{\mathrm{ra}}) = \lambda_{\mathrm{ra}}^3 + \alpha_2\lambda_{\mathrm{ra}}^2 + \alpha_1\lambda_{\mathrm{ra}} + \alpha_0 = 0\,, \tag{45}$$

with

$$\alpha_0 = -\frac{C\rho\phi\varepsilon}{2B^2\sigma_u^2}\,,$$

$$\alpha_1 = \frac{A^2\phi - AB\phi + C\varepsilon}{B^2\sigma_u^2}\,,$$

$$\alpha_2 = -\frac{\rho\phi}{2}\,,$$

$$q = \frac{3\alpha_1 - \alpha_2^2}{9}\,,$$

$$r = \frac{\alpha_1\alpha_2 - 3\alpha_0}{6} - \frac{\alpha_2^3}{27}\,.$$

If $q^3 + r^2 < 0$, there are three distinct real roots. But since $f(0) = \alpha_0 < 0$ and $f'(0) = \alpha_1 < 0$, two of them are negative. We discard them, due to the second order condition and we choose the unique positive one. If $q^3 + r^2 > 0$, then there are two complex roots (with negative real parts) and one real positive root. We choose that positive root for the value of λ_{ra}. $\square$

References

Admati, A.R., Pfleiderer, P., 1988. A theory of intraday patterns: volume and price variability. Review of Financial Studies 1, 3–40.

Alpert, M., Raiffa, H., 1982. A progress report on the training of probability assessors. In: Kahneman, D., Slovic, P., Tversky, A. (Eds.), Judgment Under Uncertainty: Heuristics and Biases. Cambridge University Press, Cambridge.

Benos, A.V., 1993. Overconfidence and limited rationality in financial markets. Ph.D. Thesis, Graduate School of Business, Stanford University.

Black, F., 1986. Noise. Journal of Finance 41, 529–543.

Blume, L., Easley, D., 1992. Evolution and market behavior. Journal of Economic Theory 58, 9–40.

Blume, L., Easley, D., O'Hara, M., 1994. Market statistics and technical analysis: The role of volume. Journal of Finance 49, 153–181.

Brock, W.A., 1988. Nonlinearity and complex dynamics in economics and finance. In: Anderson, P.W., Arrow, K.J., Pines, D. (Eds.), The Economy as an Evolving Complex System, vol. V. Addison–Wesley, Redwood City, CA, pp. 77–97.

Brown, D.P., Jennings, R.H., 1989. On technical analysis. Review of Financial Studies 2, 527–551.

Caballé, J., Sákovics, J., 1994. Overconfident speculation with imperfect competition. Mimeo, Institut d'Análisi Económica, Barcelona.

Canning, D., 1988. Rationality and game theory when players are Turing machines. Research paper TE/88/183, London School of Economics.

DeLong, J.B., Schleifer, A., Summers, L.H., Waldmann, R.J., 1990. Noise trader risk in financial markets. Journal of Political Economy 98, 703–738.

DeLong, J.B., Schleifer, A., Summers, L.H., Waldmann, R.J., 1991. The survival of noise traders. Journal of Business 64, 1–19.

Dixit, A., 1979. A model of duopoly suggesting a theory of entry barriers. Bell Journal of Economics 10, 20–32.

Figlewski, S., 1978. Market 'efficiency' in a market with heterogeneous information. Journal of Political Economy 86, 581–597.

Fishman, M.J., Hagerty, K.M., 1995. The incentive to sell financial market information. Journal of Financial Intermediation 4, 95–115.

Gilboa, I., Schmeidler, D., 1993. Updating ambiguous beliefs. Journal of Economic Theory 59, 33–49.

Griffin, D., Tversky, A., 1992. The weighing of evidence and the determinants of confidence. Cognitive Psychology 24, 411–435.

Grossman, S.J., Stiglitz, J., 1980. On the impossibility of informationally efficient markets. American Economic Review 70, 393–408.

Grundy, B., McNichols, M., 1989. Trade and the revelation of information through prices and direct disclosure. Review of Financial Studies 2, 495–526.

Jaffe, J.F., Winkler, R.L., 1976. Optimal speculation against an efficient market. Journal of Finance 31, 49–61.

Keynes, J.M., 1964. The General Theory of Employment, Interest, and Money. Harcourt, Brace & World, New York, NY.

Kreps, D.M., 1990. A Course in Microeconomic Theory. Princeton University Press, Princeton, NJ.
Kyle, A.S., 1985. Continuous auctions and insider trading. Econometrica 53, 1315–1335.
Kyle, A.S., 1989. Informed speculation with imperfect competition. Review of Economic Studies 56, 317–356.
Kyle, A.S., Wang, F.A., 1997. Speculation duopoly with agreement to disagree, Journal of Finance, forthcoming.
Kurz, M., 1994. On the structure and diversity of rational beliefs. Economic Theory 4, 877–900.
Lichtenstein, S., Fischhoff B., Phillips, L., 1982. Calibration of probabilities: The state of art to 1980. In: Kahneman, D., Slovic P., Tversky, A. (Eds.), Judgment Under Uncertainty: Heuristics and Biases. Cambridge University Press, Cambridge, UK.
March, J., Shapira, Z., 1987. Managerial perspectives on risk and risk taking. Management Science 33, 1404–1418.
Odean, T., 1996. Volume, volatility, price and profit when all traders are above average. Mimeo, Haas School of Business, UC Berkeley.
Palomino, F., 1996. Noise trading in small markets. Journal of Finance, 1537–1550.
Rubinstein, A., 1986. Finite automata play the repeated prisoners' dilemma. Journal of Economic Theory 39, 83–96.
Schaffer, M., 1989. Are profit maximisers the best survivors?. The Journal of Economic Behaviour and Organization 12, 29–45.
Subrahmanyam, A., 1991. Risk aversion, market liquidity and price efficiency. Review of Financial Studies 4, 417–441.
Tversky, A., Kahneman, D., 1974. Judgment under uncertainty: heuristics and biases. Science 185, 1124–1131.
Vickers, J., 1985. Delegation and the theory of the firm. The Economic Journal 95, 138–147.